Probabilistic Graphical Models

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Exercise Sheet 2 v1.0

1 Probabilties III

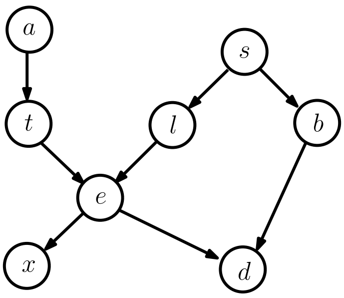
2 The Monty Hall Problem

The following famous problem is known as the Monty Hall problem. In a game show a candidate is offered to choose from three doors. Behind one door there is a prize, behind the other two doors there is a goat. The candidate receives whatever is behind the door of her choice and is more interested in the prize than the goat.

After the candidates chooses a door and informs the game-show host about it, the host opens one from the other two doors that do **not** contain the prize. Now the candidate is offered to change her choice, she can again choose freely from the two remaining doors. Should she stick to her first choice or change to the other remaining door?

Imagine you are a candidate and recently took a course on probabilistic graphical models. Model the problem as two Bayesian networks, one in which you switch and one where you don't: introduce suitable random variables, a suitable network structure and specify the prior/conditional probabilities. Then, compute the probability of winning the prize in each of the two cases. *(Note: this is an exercise about Bayesian networks. Even if you know the problem and its solution already, please follow the above path and do not, e.g., argue combinatorically, etc.)*

3 Chest Clinic Network



The chest clinic network above concerns the diagnosis of lung disease (tuberculosis,lung cancer, or both, or neiter). In this model a visit to asia is assumed to increase the probability of tuberculosis. We have the following binary variables.

x	positive X-ray
d	Dyspnea (shortness of breath)
e	Either Tuberculosis or Lung Cancer
t	Tuberculosis
l	Lung cancer
b	Bronchitis
a	Visited Asia
s	Smoker

1. Write down the factorization of the distribution implied by the graph.

2. Are the following independence statements implied by the graph? (And how do you conclude this?)

   (a) tuberculosis$\perp\!\!\!\perp$smoking|shortness of breath

   (b) lung cancer$\perp\!\!\!\perp$bronchitis|smoking

   (c) visit to Asia$\perp\!\!\!\perp$smoking|lung cancer

   (d) visit to Asia$\perp\!\!\!\perp$smoking|lung cancer,shortness of breath

3. Calculate by hand the values for $p(d)$. The Conditional Probability Table (CPT) is:

$$\begin{array}{lclclcl}
p(a=1) & = & 0.01, & p(s=1) & = & 0.5 \\
p(t=1 \mid a=1) & = & 0.05, & p(t=1 \mid a=0) & = & 0.01 \\
p(l=1 \mid s=1) & = & 0.1, & p(l=1 \mid s=0) & = & 0.01 \\
p(b=1 \mid s=1) & = & 0.6, & p(b=1 \mid s=0) & = & 0.3 \\
p(x=1 \mid e=1) & = & 0.98, & p(x=1 \mid e=0) & = & 0.05 \\
p(d=1 \mid e=1, b=1) & = & 0.9, & p(d=1 \mid e=1, b=0) & = & 0.7 \\
p(d=1 \mid e=0, b=1) & = & 0.8, & p(d=1 \mid e=0, b=0) & = & 0.1
\end{array}$$

   and

$$p(e=1 \mid t, l) = \begin{cases} 0 & t=0 \wedge l=0 \\ 1 & \text{otherwise} \end{cases}$$

   Important: Think before you start calculating; what is a good sequence of calculations?

4 Markov Equivalence of Graphs

The following two adjacency matrices specify two directed graphs, using the convention that for $A = (a_{ij})_{i,j=1,\ldots,9}$, an entry $a_{ij} = 1$ means that node i is a parent of node j.

$$A_1 = \begin{pmatrix}
0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \qquad
A_2 = \begin{pmatrix}
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

a) Draw the corresponding graphs (by hand, or using a suitable software).

b) Are they Markov equivalent?

c) For each graph, construct the smallest *undirected* graph that can represent all distributions (though potentially more) as the undirected ones.

5 Limits of Bayesian Networks

The graph structure of a Bayesian network encodes some independence relations, but not all are possible. Can you construct a class of distributions with a set of (conditional) independence relations that cannot be represented by a Bayesian network?